REMARKS

Claims 1-41 are pending in this application and stand rejected. No claims have been amended in this response and no claims have been added. The specification has been amended to add paragraph numbers. No new matter has been added. Applicant respectfully requests reconsideration of this application and its pending claims in light of the following remarks.

I. Drawings

Applicant respectfully submits formal drawings (Figures 1-43) herewith in compliance with 37 CFR 1.84(f).

II. Request for Information

Applicant respectfully submits of the following publications as cited in the instant application for the Examiner's review.

> "Fundamentals of Geostatics in Five Lessons", Journel, A.G., Short Course in Geology, Vol. 8, 44 pp. 15-16, AGU, Washington, D.C., 1989.
>
> "GSLIB Geostatistical Software Library and User's Guide second edition", Deutsch, Clayton V. and Journel, A., Oxford University Press, pp. 14-15, New York, Oxford, 1998.

III. The Specification

A substitute specification is submitted herewith, amended to include paragraph numbering. No amendments have been made. No new matter has been added.

IV. The § 103 Rejections

In the Office Action dated April 8, 2002, the Examiner rejected claims 1-41 under 35 USC 103(a) as being unpatentable.

A. Claim 1

Claim 1 is rejected under 35 U.S.C. 103(a) as being unpatentable over Jones in view of Matteucci and Tucker. Applicant respectfully traverses this rejection.

Claim 1 recites:

> A method of generating a map illustrating a set of characteristics of a cross section through an earth formation representing a time slice or a horizon through said formation in response to a plurality of scattered data observations on said cross section representing a plurality of parameters located at a plurality of locations on said cross section, comprising the steps of:
> (a) gridding said cross section thereby generating a gridded cross section which includes a grid having a plurality of intersections and said plurality of scattered data observations distributed among the intersections of said grid on said cross section;
> (b) obtaining a unique cumulative distribution function associated with each intersection of the grid of the gridded cross section thereby producing a plurality of cumulative distribution functions associated, respectively, with the plurality of intersections of said grid;
> (c) choosing a value from each of the cumulative distribution function at each of the intersections of the gridded cross section thereby producing a plurality of values associated, respectively, with the plurality of intersections, and
> (d) assigning each value to its associated intersection of the gridded cross section and assigning a unique color to said each value thereby generating a map illustrating said set of characteristics of said cross section through said earth formation.

The Examiner states that **Jones** (U.S. Patent No. 5,838,634) discloses "a gridded cross section" with its disclosure of a three dimensional array of cells and mention of "structural surfaces or horizons in the form of 2-D computer grids or meshes." Applicant agrees that **Jones** discloses 3-D arrays of cells, which are known in the art and the 3D array inherently includes cross sections of horizons. As the Examiner points out, **Jones**

mentions modeling horizons or structural surfaces with meshes or grids. Applicant notes that **Jones** does not mention modeling time slices with grids and **Jones** is not directed to any further analysis with respect to gridded cross sections. Instead, **Jones** divides the formation up into blocks (not cross sections) and the modeling described by **Jones** is performed on a block basis, *see* **Jones** column 12, lines 46 through column 13, line 29, not through the use of cross sections. Although grids by definition have intersections, Jones thus does not disclose "said plurality of scattered data observations distributed among the intersections of said grid on said cross section… " or the other limitations of claim 1, as acknowledged by the Examiner.

Matteucci (U.S. Patent No. 5,884,229) "is a statistical method for quantitatively measuring the lateral continuity of the seismic reflection character at in specified location in a subsurface target formation." (Mateucci, column 3, lines 35-38.) In **Mateucci**, the analysis is focused on a comparison of seismic traces, not 3-D arrays or gridded cross sections. A seismic data trace is selected as a "reference trace." Note that each seismic trace is a response to sound waves bouncing off underground formations and can be seen as a vertical, wavy line **20**, passing through the formation in the z direction in **Mateucci**, **FIG. 1**. In **Mateucci**, "all [seismic] traces falling within a specified distance from the reference trace are extracted from the survey, and one or more statistics which measure the similarity between the reference trace and each of the other traces are calculated. … Finally, the calculated statistics are used to make a determination of whether or not the reflection character within the target interval of each of the extracted traces is the same as or different from the reference trace." (Mateucci, column 5, lines 29-44). While **Mateucci** uses the cumulative distribution function statistical tool, it is within the context

of this comparison of seismic traces and each cumulative distribution function is taken for a particular seismic trace (*see* Mateucci column 7, lines 2-15). This is a wholly different context than that of the present invention, where the cumulative distribution functions are taken at each intersection of the grid on the cross section. **Mateucci** goes on to take the Kolmogorov-Smirnov statistic, which "measures the similarity of the amplitude distribution of the two seismic traces." (Column 6 line 58- 60 and continuing through column 7, line 15). Again, **Mateucci** is using the cumulative distribution functions to measure the similarity of two vertical seismic traces, not investigating similarity of points along a grid on a cross section through the formation.

If one were to combine **Jones** and **Mateucci**, the result would be a 3-D cube array or block, having horizons gridded but with no further analysis of the gridded horizons. The array would have a plurality of seismic traces passing vertically through it and the invention would involve an analysis of the similarity of seismic traces using cumulative distribution functions and Kolmogorov-Smirnov statistics. This does not meet the limitations as recited by claim 1, which include "a plurality of scattered data observations distributed among the intersections of said grid on said cross section; obtaining a unique cumulative distribution function associated with each intersection of the grid … choosing a value from each of the cumulative distribution function at each of the intersections of the gridded cross section … and assigning each value to its associated intersection of the gridded cross section and assigning a unique color to said each value …" (Emphasis added.)

Nor does **Tucker** (The Computer Science and Engineering Handbook, edited by Allen B. Tucker, Jr., 1997) supply the deficiencies of **Jones** and **Mateucci**. While

Tucker does disclose the use of color in "a particular slice of 3-D dataset," it does not disclose the use of color as recited in claim 1, i.e., assigned to a value taken from the cumulative distribution function of an intersection on the grid of the cross section.

Accordingly, the combination of **Jones**, **Mateucci** and **Tucker** does not disclose "a plurality of scattered data observations distributed among the intersections of said grid on said cross section; obtaining a unique <u>cumulative distribution function</u> <u>associated with each intersection of the grid;</u> ... choosing a value from each of the cumulative distribution function <u>at each of the intersections</u> of the gridded cross section; ... and assigning each value to its associated intersection of the gridded cross section and assigning a unique color to said each value" as recited by claim 1 (emphasis added). Thus claim 1 as amended is felt to distinguish patentably over the combination of the **Jones**, **Mateucci** and **Tucker** references.

B. Claim 2

Claim 2 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** (Fundamentals of Geostatics in Vive Lessons," by Journel, vol. 8AGU, 1989). Applicant respectfully traverses this rejection.

As the Examiner mentions, claim 2 depends from claim 1 with three additional limitations, which are not disclosed by **Jones**: (b1) Krigging the gridded cross section thereby generating a plurality of expected values and a plurality of corresponding standard deviations <u>associated, respectively, with the plurality of intersections of the grid</u> of the gridded cross section; (b2) producing a probability density function associated with each expected value and each corresponding standard deviation generated from step (b1) thereby producing <u>a plurality of probability density functions corresponding,</u>

respectively, to the plurality of intersections of the grid of the gridded cross section; and (b3) producing a cumulative distribution function associated with each probability density function produced from step (b2) thereby producing a plurality of cumulative distribution functions corresponding, respectively, to the plurality of probability density functions which correspond, respectively, to the plurality of intersections of the grid of the gridded cross section.

Claim 2 is dependent on claim 1 and contains all of its limitations so it, too, is felt to be distinguishable from the cited references. With respect to its additional limitations, the Examiner states that Krigging and probability functions are described in the instant Specification, which is correct, but the use of such tools as recited by claim 2 in connection with being taken at and corresponding to intersections of the gridded cross section as recited by claim 2. Similarly, as previously pointed out, Matteucci discloses the use of cumulative distribution functions, but for use in comparing seismic traces, not taken at intersections of grids on gridded cross sections. Accordingly, claim 2 is felt to be patentably distinguishable from the cited references.

C. Claim 3

Claim 3 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Hogg** (Probability and Statistical Inference, by Hogg, Robert V and Tanis, Eliot A, 3d ed. 1988). Claim 3 is dependent on claim 1 and contains all of its limitations so it, too, is felt to be distinguishable from the cited references. The addition of **Hogg** does not affect this as the reference does not disclose the application of the statistical tool of choosing a probability from the cumulative distribution function in the circumstance where the cumulative distribution

function is taken at an intersection of a grid on a gridded cross section. Such an application is not disclosed by **Hogg**, which does not supply the deficiencies of the previously discussed references. Accordingly, Applicant respectfully traverses this rejection.

D. Claim 4

Claim 4 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Hogg**. Applicant respectfully traverses this rejection. Claim 4 depends from claim 2, contains all its limitations and is felt to be likewise patentable over the cited references. As described with respect to claim 3, the addition of Hogg does not supply the deficiencies of the previously discussed references. Therefore, Applicant respectfully asserts it has traversed this rejection.

E. Claim 5

Claim 5 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Hogg**. Claim 5 depends from claim 2, contains all its limitations and is felt to be likewise patentable over the cited references. As described with respect to claim 3, the addition of Hogg does not supply the deficiencies of the previously discussed references. Therefore, Applicant respectfully asserts it has traversed this rejection.

F. Claim 6

Claim 6 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Hogg.** Claim 6 depends from claim 2, contains all its limitations and is felt to be likewise patentable over the cited references. As described with respect to claim 3, the addition of Hogg does not supply the

deficiencies of the previously discussed references. Therefore, Applicant respectfully asserts it has traversed this rejection.

G. Claim 7

Claim 7 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Hogg**. Claim 7 depends from claim 2, contains all its limitations and is felt to be likewise patentable over the cited references. As described with respect to claim 3, the addition of Hogg does not supply the deficiencies of the previously discussed references. Therefore, Applicant respectfully asserts it has traversed this rejection.

H. Claim 8

Claim 8 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Webber** (U.S. Patent No. 6,081,577). Claim 8 depends from claim 2 and contains all of its limitations, so it is felt to be likewise patentable over the cited references. Webber does not supply the deficiencies of the previously discussed references. While it does, as the Examiner points out, disclose the use of affine correction, it is in the context of the transformation of three dimensional matrix volumes and des not disclose "applying an affine correction to each of the values chosen from each of the cumulative distribution functions associated with each of the intersections of the gridded cross section" as recited by claim 8. Therefore, Applicant respectfully asserts that this rejection is traversed.

I. Claim 9

Claim 9 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and **Journel** and **Webber**. Claim 9 depends from claim

8 and contains all of its limitations, so it is felt to be likewise patentable over the cited references. Therefore, Applicant respectfully asserts that this rejection is traversed.

J. Claims 10- 14

Claims 10-14 are rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteuci** and **Tucker** and **Journel** and **Hogg**. Claims 10-14 depend from claim 9, contain all of its limitations, and so are felt to be likewise patentable over the cited references. Therefore, Applicant respectfully asserts that this rejection is traversed.

K. Claim 15

Claim 15 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker,** for the reasons described with respect to claim 1. Claim 15 is an independent "Program storage device ... cross section claim" with limitations corresponding to those of claim 1. Therefore, claim 15 is felt to be patentably distinguishable from the cited references for the same reasons discussed above with respect to claim 1, i.e. that the cited references do not disclose or suggest "obtaining a unique cumulative distribution function <u>associated with each intersection of the grid</u> of the gridded cross section thereby producing a plurality of cumulative distribution functions associated, respectively, with the plurality of intersections of said grid" or "choosing a value from each of the cumulative distribution function at each of the intersections of the gridded cross section ...;" or "assigning each value to its associated intersection of the gridded cross section and assigning a unique color to said each value...." Therefore, Applicant respectfully asserts that this rejection is traversed.

L. Claims 16-24

Claims 16-24 are rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker**, and, variously, in combination with **Journel**, **Hogg** and **Webber** for the reasons described with respect to claims 2-14. Claims 16-24 are dependent, directly or indirectly, on claim 15 and contain all of its limitations. For the reasons described above with respect to claim 15 and claims 2-14, it is believed that claims 16-24 are patentably distinguishable from the cited references and that this rejection is traversed.

M. Claim 25

Claim 25 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker**. Claim 25 is an independent apparatus claim with limitations corresponding to those of claim 1. Claim 25 is felt to be patentably distinguishable from the cited references for the same reasons discussed above with respect to claim 1, i.e. that the cited references do not disclose or suggest " a second apparatus responsive to said first gridded cross section adapted for Kriging said first gridded cross section thereby generating a second gridded cross section having a plurality of intersections wherein each intersection of said second gridded cross section includes an expected value of a parameter and a standard deviation; third apparatus responsive to said second gridded cross section for generating a plurality of cumulative distribution functions associated, respectively, with said plurality of intersections of said second gridded cross section; and fourth apparatus adapted for selecting a plurality of values, respectively, from said plurality of cumulative distribution functions and for assigning said plurality of values and a plurality of

unique colors to the respective plurality of intersections of said second gridded cross section thereby generating said map." Therefore, Applicant respectfully asserts that this rejection is traversed.

N. Claims 26-29

Claims 26-29 are rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and, variously, in combination with **Journel**, **Hogg** and **Webber** for the reasons described with respect to claims 2-14. Claims 26-29 are dependent, directly or indirectly, on claim 25 and contain all of its limitations. For the reasons described above with respect to claim 25 and claims 2-14, it is believed that claims 26-29 are patentably distinguishable from the cited references and that this rejection is traversed.

O. Claim 30

Claim 30 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** for the reasons described with respect to claim 1. Claim 30 recites:

> A method of generating a cube illustrating a set of characteristics of an earth formation disposed within a cubic volume of earth, said cube including a plurality of cross sections, each cross section including a plurality of scattered data samples, each cross section being gridded and including a plurality of intersections, comprising the steps of:
> (a) determining a plurality of cumulative distribution functions corresponding, respectively, to the plurality of intersections for each of said plurality of cross sections;
> (b) selecting a value from each of said cumulative distribution functions thereby selecting a plurality of values corresponding, respectively, to said plurality of cumulative distribution functions for each of said plurality of cross sections;

> (c) assigning said plurality of values, respectively, to said plurality of intersections for each of said plurality of cross sections; and
> (d) assigning a plurality of unique colors, respectively, to said plurality of values assigned, respectively, to said plurality of intersections.

As described in this response to claim 1, the cited references do not disclose "determining a plurality of <u>cumulative distribution functions corresponding, respectively, to the plurality of intersections for each of said plurality of cross sections</u>," or "selecting a value from each of said cumulative distribution functions thereby selecting a plurality of values corresponding, respectively, to said plurality of cumulative distribution functions for each of said plurality of cross sections;" or "assigning said plurality of values, respectively, to said plurality of intersections for each of said plurality of cross sections," or "assigning a plurality of unique colors, respectively, to said plurality of values assigned, respectively, to said plurality of intersections." Therefore, Applicant respectfully asserts that this rejection is traversed.

P. Claims 31-35

Claims 31-35 are rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and, variously, in combination with **Journel**, **Hogg** and **Webber** for the reasons described with respect to claims 2-14. Claims 31-35 are dependent, directly or indirectly, on claim 30 and contain all of its limitations. For the reasons described above with respect to claim 30 and claims 2-14, it is believed that claims 31-35 are patentably distinguishable from the cited references and that this rejection is traversed.

Q. Claim 36

Claim 36 is rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker**. Claim 36 is directed to a "program storage device readable by a machine, tangibly embodying a program of instructions executable by the machine to perform method steps for generating a cube..." and stands rejected for the reasons described with respect to claim 1.

As described in this response to the rejections cited against claim 1, the cited references do not disclose "determining a plurality of cumulative distribution functions corresponding, respectively, to the plurality of intersections for each of said plurality of cross sections;" or "selecting a value from each of said cumulative distribution functions thereby selecting a plurality of values corresponding, respectively, to said plurality of cumulative distribution functions for each of said plurality of cross sections;" or "assigning said plurality of values, respectively, to said plurality of intersections for each of said plurality of cross sections;" or "assigning a plurality of unique colors, respectively, to said plurality of values assigned, respectively, to said plurality of intersections." Therefore, Applicant respectfully asserts that this rejection is traversed.

R. Claims 37-41

Claims 37-41 are rejected under 35 U.S.C. 103(a) as being unpatentable over **Jones** in view of **Matteucci** and **Tucker** and, variously, in combination with **Journel**, **Hogg** and **Webber** for the reasons described with respect to claims 2-14. Claims 37-41 are dependent, directly or indirectly, on claim 36 and contain all of its limitations. For the reasons described above with respect to claim 36 and claims 2-14, it is believed that

claims 37-41 are patentably distinguishable from the cited references and that this rejection is traversed.

CONCLUSION

It is respectfully submitted that this application, as now amended, is in condition for allowance for the reasons stated above. Applicant respectfully requests reconsideration of this application and allowance of all its pending claims

This amendment is intended to be a complete response to the Office Action dated April 8, 2003.

Respectfully submitted,

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Date: October 8, 2003

Enclosures:

1. Transmittal Form.
2. Fee Transmittal and Authorization to Charge Deposit of Account.
3. Fee Determination Record.
4. Petition for Extension of Time and Authorization to Charge Deposit Account.
5. Formal Drawings (Figs. 1-43)
6. Copies of Examiner's Requested Publications:

   "Fundamentals of Geostatics in Five Lessons", Journel, A.G., Short Course in Geology, Vol. 8, 44 pp. 15-16, AGU, Washington, D.C., 1989.

   "GSLIB Geostatistical Software Library and User's Guide second edition", Deutsch, Clayton V. and Journel, A., Oxford University Press, pp. 14-15, New York, Oxford, 1998.
7. Acknowledgment Postcard.

Short Course in Geology: Volume 8

PART OF #6

Fundamentals of Geostatistics in Five Lessons

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American Geophysical Union

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Short Course Series Editors

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$$Z^*(x) = b_0 + \sum_{\alpha=1}^{n} b_\alpha f(x_\alpha - x) \quad (30)$$

with:

$$Z^*(x_\alpha) = b_0 + \textstyle\sum_{\beta=1}^{n} b_\beta f(x_\beta - x_\alpha) = \text{ datum } z(x_\alpha) \quad \text{for all } \alpha = 1, \cdots, n \quad (31)$$

Then one may argue on the arbitrariness of choosing, in the SK case, covariance functions for interpolation functions. As was shown before, the choice of covariance functions allows to minimize the resulting error variance, $\text{Var}\{Z_0 - Z_0^*\}$.

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Lesson III: Linear Regression Under Constraint(s) and Ordinary Kriging (OK)

In all the simple kriging (SK) developments of the previous Lesson II, the mean(s) were supposedly known, cf. relations (9) and (18). Consider, for example, the SK estimator (18) of an unknown $Z(x_0)$ from n data related to the same attribute but at different locations $x_\alpha, \alpha = 1, \cdots, n$:

$$Z^*(x_0) - m = \sum_{\alpha=1}^{n} \lambda_\alpha [Z(x_\alpha) - m] \quad (1)$$

The common (stationary) mean m is supposed known. If the process $Z(x)$ was sampled at all locations x within the formation A considered, that mean m would be known but also there would not be any estimation problem left. Thus, in practice, either m is to be estimated prior to the SK algorithm, or an estimation algorithm should be designed which does not require prior estimate of that mean.

Consider the linear estimator:

$$Z^*(x_0) = \lambda_0 + \sum_{\alpha=1}^{n} \lambda_\alpha Z(x_\alpha)$$

The error mean is:

$$E\{Z(x_0) - Z^*(x_0)\} = m - \lambda_0 - \sum_{\alpha=1}^{n} \lambda_\alpha m = -\lambda_0 + m\left(1 - \sum_\alpha \lambda_\alpha\right)$$

and should be set to zero, whatever the unknown value of m. This can be achieved only if:

$$\begin{cases} \lambda_0 = 0 \\ \sum_{\alpha=1}^{n} \lambda_\alpha = 1 \end{cases}$$

Thus, an unbiased linear estimator of $Z(x_0)$ is written as:

$$Z^*(x_0) = \sum_{\alpha=1}^{n} \lambda_\alpha Z(x_\alpha) \ , \ \text{with: } \sum_{\alpha=1}^{n} \lambda_\alpha = 1 \quad (2)$$

Note. Although for reason of convenience, we have used the same notation λ_α for the SK weights of relation (1) and the weights of relation (2) they are not equal.

The error variance is written, as in Lesson II (11):

$$\sigma_E^2 = \text{Var}\{Z(x_0) - Z^*(x_0)\} = \sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} a_\alpha a_\beta C(x_\alpha - x_\beta) \quad (3)$$

with: $\begin{cases} a_0 = 1 \\ a_\alpha = -\lambda_\alpha, \alpha = 1, \cdots, n \end{cases}$

and: $C(x_\alpha - x_\beta) = \text{Cov}\{Z(x_\alpha), Z(x_\beta)\}$

We wish to minimize that error variance, still ensuring unbiasedness f the estimator, i. ., ensuring the constraint: $\sum_{\alpha=1}^{n} \lambda_\alpha = 1$. This amounts to an optimization under linear constraint. The Lagrange formalism will be used whereby a function, of the $(n+1)$ parameters

$\lambda_\alpha, \alpha = 1, \cdots, n$ and 2μ being th Lagrange parameter, is defined as:

$$S(\lambda_\alpha, \alpha = 1, \cdots, n\,;\, \mu) = \sigma_E^2 + 2\mu \left[\sum_{\alpha=1}^{n} \lambda_\alpha - 1\right]$$

The extreme of that function S is obtained by setting to zero its $(n+1)$ partial derivatives:

$$-\tfrac{1}{2}\tfrac{\partial S}{\partial \lambda_\alpha} = \textstyle\sum_{\beta=0}^{n} a_\beta C(x_\alpha - x_\beta) - \mu = 0 \ , \quad \alpha = 1, \cdots, n$$

$$-\tfrac{1}{2}\tfrac{\partial S}{\partial \mu} = \textstyle\sum_{\alpha=1}^{n} \lambda_\alpha - 1 = 0$$

Th n first equations are rewritten:

$$C(x_\alpha - x_0) - \sum_{\beta=1}^{n} \lambda_\beta C(x_\alpha - x_\beta) - \mu = 0 \text{, i.e.,}$$

$$\sum_{\beta=1}^{n} \lambda_\beta C(x_\alpha - x_\beta) + \mu = C(x_\alpha - x_0) \ , \ \alpha = 1, \cdots, n.$$

Finally, the $(n+1)$ unknowns, the λ_α's and μ, are given by a system of $(n+1)$ linear equations, known as the "constrained normal" system of equations, belatedly renamed "ordinary kriging" (OK) system:

$$\begin{cases} \sum_{\beta=1}^{n} \lambda_\beta C(x_\alpha - x_\beta) + \mu = C(x_\alpha - x_0) \ , \ \alpha = 1, \cdots, n \\ \sum_{\beta=1}^{n} \lambda_\beta = 1 \end{cases} \tag{4}$$

The corresponding minimized error variance, also called "ordinary kriging" (OK) variance is written:

$$\begin{aligned} \sigma_{OK}^2 &= E\{[Z(x_0) - Z^*(x_0)]^2\} = \\ &= C(0) - \textstyle\sum_{\alpha=1}^{n} \lambda_\alpha C(x_\alpha - x_\beta) - \mu \geq 0 \end{aligned} \tag{5}$$

Just like the SK system of Lesson II (12), the OK system (4) presents one and only one solution as soon as the data covariance matrix $\underline{K} = [C(x_\alpha - x_\beta)]$ is positive definite, in practice as soon as:

(i) the covariance function $C(h)$ is licit, i.e., is a positive definite function, see Lesson I condition (34)

(ii) there are no two data locations totally redundant, i.e.,

$$C(x_\alpha - x_\beta) = C(x_{\alpha'} - x_\beta) \text{ , for all } \beta = 1, \cdots, n \text{ , if and only if: } \alpha = \alpha'$$

Independence case

If the n RV data $Z(x_\alpha)$ are independent (or uncorrelated) one from another, the data covariance matrix $[C(x_\alpha - x_\beta)]$ reduces to a diagonal matrix with all elements of the diagonal equal to the common variance $C(0) = \text{Var}\,\{Z(x)\}$, for all x.
The OK system (4) becomes:

$$\begin{cases} \lambda_\alpha C(0) + \mu = C(x_\alpha - x_0) \ , \ \alpha = 1, \cdots, n \\ \sum_\alpha \lambda_\beta = 1 \end{cases} \tag{6}$$

i.e.

$$\begin{cases} \lambda_\alpha = \frac{C(x_\alpha - x_0)}{C(0)} - \frac{\mu}{C(0)} = \rho(x_\alpha - x_0) - \frac{\mu}{C(0)} \\ -\mu = \frac{C(0)}{n}\left[1 - \sum_{\alpha=1}^{n} \rho(x_\alpha - x_0)\right] \end{cases}$$

$$\begin{aligned} \sigma_{OK}^2 &= C(0)\left[1 - \textstyle\sum_{\alpha=1}^{n} \rho^2(x_\alpha - x_0)\right] \\ &\quad -\mu\left[1 - \textstyle\sum_{\alpha=1}^{n} \rho(x_\alpha - x_0)\right] \geq 0 \end{aligned}$$

with: $\rho(h) = C(h)/C(0) \in [-1, +1]$ being the standardized covariance, or "correlogram," measuring the correlation between two values $Z(x), Z(x+h)$ distant of vector h.

If, moreover, the n data are uncorrelated with the unknown RV $Z(x_0)$:
$\rho(x_\alpha - x_0) = 0$, $\alpha = 1, \cdots, n$, the OK system (6) then yields:

$$\begin{cases} \lambda_\alpha = 1/n \ , \ \alpha = 1, \cdots, n \\ -\mu = C(0)/n \end{cases} \tag{7}$$

Thus: $Z^*(x_\alpha) = \frac{1}{n}\sum_{\alpha=1}^{n} Z(x_\alpha)$, whatever the location x_0 and: $\sigma_{OK}^2 = C(0) + \frac{C(0)}{n} \geq 0$

In the case of total spatial independence, the OK estimate reduces to the arithmetic mean of the n data retained.

In the SK case Lesson II (16), the SK estimate did reduce to the known mean m_0. In the OK case, that mean is unknown and is estimated by the mean of the n data.

The exactitude property

If one datum, say, $Z(x_{\alpha'})$, is considered to be the unknown $Z(x_0)$, then:
$C(x_{\alpha'} - x_\beta) = C(x_0 - x_\beta)$, for all $\beta = 1, \cdots, n$, and the OK system (4) is written:

$$\begin{cases} \sum_{\substack{\beta=1 \\ \beta \neq \alpha'}}^{n} \lambda_\beta C(x_\alpha - x_\beta) + \lambda_{\alpha'} C(x_\alpha - x_0) + \mu = C(x_\alpha - x_0) \ , \\ \alpha = 1, \cdots, n \\ \sum_{\beta=1}^{n} \lambda_\beta = 1 \end{cases}$$

The unique solution is: $\lambda_{\alpha'} = 1, \lambda_\beta = 0 \beta \neq \alpha', \mu = 0$.
Thus, the OK estimate identifies the datum value $Z(x_{\alpha'}) = Z(x_0)$, and:

$$\sigma_{OK}^2 = C(0) - \lambda_{\alpha'} C(0) = 0 \tag{8}$$

As SK, the OK algorithm features the exactitude property: the OK surface, $Z^*(x_0)$, $x_0 \in$ Domain, honors the data values at data locations.

PART OF #6

GSLIB

Geostatistical Software Library and User's Guide

Second Edition

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*semi*variogram models, which are then promptly converted into equivalent covariance models. The "variogram" programs of Chapter III allow the computation of covariance functions in addition to variogram functions.

A Note on Generalized Covariances

Generalized covariances of order k are defined as variances of differences of order $(k+1)$ of the initial RF $Z(\mathbf{u})$ (see, e.g. [39, 187]). The traditional variogram $2\gamma(\mathbf{h})$, defined in relation (II.11) as the variance of the first-order difference of the RF $Z(\mathbf{u})$, is associated to a generalized covariance of order zero. The order zero stems from the variogram expression's filtering any zero-order polynomial of the coordinates $\mathbf{u}$, such as $m(\mathbf{u})$=constant, added to the RF model $Z(\mathbf{u})$. Similarly, a generalized covariance of order k would filter a polynomial trend of order k added to the RF model $Z(\mathbf{u})$.

Unfortunately, inference of generalized covariances of order $k > 0$ poses severe problems since experimental differences of order $k+1$ are not readily available if the data are not gridded. In addition, more straightforward algorithms for handling polynomial trends exist, including ordinary kriging with moving data neighborhoods [112].

Therefore, notwithstanding the possible theoretical importance of the IRF-k formalism, it has been decided not to include generalized covariances and the related intrinsic RF models of order k in this version of GSLIB.

II.1.4 Kriging

Kriging is "a collection of generalized linear regression techniques for minimizing an estimation variance defined from a prior model for a covariance" ([146], p. 41).

Consider the estimate of an unsampled value $z(\mathbf{u})$ from neighboring data values $z(\mathbf{u}_\alpha), \alpha = 1, \ldots, n$. The RF model $Z(\mathbf{u})$ is stationary with mean m and covariance $C(\mathbf{h})$. In its simplest form, also known as simple kriging (SK), the algorithm considers the following linear estimator:

$$Z^*_{SK}(\mathbf{u}) = \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u}) Z(\mathbf{u}_\alpha) + \left(1 - \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u})\right) m \quad \text{(II.12)}$$

The weights $\lambda_\alpha(\mathbf{u})$ are determined to minimize the error variance, also called the "estimation variance." That minimization results in a set of normal equations [102, 124]:

$$\sum_{\beta=1}^{n} \lambda_\beta(\mathbf{u}) C(\mathbf{u}_\beta - \mathbf{u}_\alpha) = C(\mathbf{u} - \mathbf{u}_\alpha), \quad \text{(II.13)}$$

$$\forall \alpha = 1, \ldots, n$$

The corresponding minimized estimation variance, or kriging variance, is:

$$\sigma^2_{SK}(\mathbf{u}) = C(0) - \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u}) C(\mathbf{u} - \mathbf{u}_\alpha) \geq 0 \qquad \text{(II.14)}$$

Ordinary kriging (OK) is the most commonly used variant of the previous simple kriging algorithm, whereby the sum of the weights $\sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u})$ is constrained to equal 1. This allows building an estimator $Z^*_{OK}(\mathbf{u})$ that does not require prior knowledge of the stationary mean m, yet remains unbiased in the sense that $E\{Z^*_{OK}(\mathbf{u})\} = E\{Z(\mathbf{u})\}$.

Non-linear kriging is but linear kriging performed on some non-linear transform of the z-data, e.g., the log-transform $\ln z$ provided that $z > 0$, or the indicator transform as defined in relation (II.6).

Traditionally, kriging (SK or OK) has been performed to provide a "best" linear unbiased estimate (BLUE) for unsampled values $z(\mathbf{u})$, with the kriging variance being used to define Gaussian-type confidence intervals, e.g.,

$$\text{Prob}\{Z(\mathbf{u}) \in [z^*_{SK}(\mathbf{u}) \pm 2\sigma_{SK}(\mathbf{u})]\} \cong 0.95$$

Unfortunately, kriging variances of the type (II.14), being independent of the data values, only provides a comparison of alternative geometric data configurations. Kriging variances are usually not measures of local estimation accuracy [99].

In addition, users have come to realize that kriging estimators of type (II.12) are "best" only in the least-squares error sense for a given covariance/variogram model. Minimizing an expected squared error need not be the most relevant estimation criterion for the study at hand; rather, one might prefer an algorithm that would minimize the impact (loss) of the resulting error; see [67, 167]. This decision-analysis approach to estimation requires a probability distribution of type (II.2), $\text{Prob}\{Z(\mathbf{u}) \leq z|(n)\}$, for the RV $Z(\mathbf{u})$ [15, 102].

These remarks seem to imply the limited usefulness of kriging and geostatistics as a whole. Fortunately, the kriging algorithm has two characteristic properties that allow its use in determining posterior ccdfs of type (II.2). These two characteristic properties are the basis for, respectively, the multi-Gaussian (MG) approach and the indicator kriging (IK) approach to determination of ccdfs:

(i) **The Multi-Gaussian Approach:** If the RF model $Z(\mathbf{u})$ is multivariate Gaussian,[3] then the simple kriging estimate (II.12) and variance (II.14) identify the mean and variance of the posterior ccdf. In addition, since

[3] If the sample histogram is not normal, a normal score-transform (a non-linear transform) can be performed on the original z-data. A multi-Gaussian model $Y(\mathbf{u})$ is then adopted for the normal score data. For example, the transform is the logarithm, $y(\mathbf{u}) = \ln z(\mathbf{u})$, if the z-sample histogram is approximately lognormal. Kriging and/or simulation are then performed on the y-data with the results appropriately back-transformed into z-values (see programs **nscore** and **backtr** in Chapter VI).